Example

A picture frame is 5 inches shorter than 3 times its width. If the perimeter of the frame is 30 inches, find the dimensions of the frame.

Solution steps:

• The dimensions of the frame are height $H$ by width $W$, where we need to determine $H$ and $W$. We are given that the height is 5 inches shorter than 3 times its width, or that

$$H = 3W - 5.$$  

• Now use the second piece of information that we have been given, namely that the perimeter of the frame is 30 inches. Recall that the perimeter is the sum of the length of all of the sides or, since there are two sides of height $H$ and two sides of width $W$,

$$2H + 2W = 30.$$  

• In the last equation above, replace $H$ by $3W - 5$ so that there is only one unknown variable ($W$) remaining in the perimeter equation:

$$2(3W - 5) + 2W = 30,$$

or

$$6W - 10 + 2W = 30,$$  

or

$$8W - 10 = 30.$$  

• Add 10 to both sides of the last equation to obtain

$$8W = 30 + 10,$$  

or

$$8W = 40,$$

and divide both sides of the equation by 8 to obtain

$$W = \frac{40}{8} = 5$$

(in inches).

• Now go back and determine the height of the frame using the relationship $H = 3W - 5$ found earlier:

$$H = 3W - 1 = 3 \cdot 5 - 5 = 10$$

(in inches).

Therefore the height of the picture frame is 10 inches and the width is 5 inches.