Example

Solve the following equation for $x$:

$$\sqrt{6x + 1} + 1 = x.$$ 

Solution: This is an equation involving a radical (i.e., the square root) as well as terms not involving a radical.

1. Isolate the radical(s) on one side of the equation, with all other terms on the other side of the equation. In this case, subtract 1 from both sides of the equation to obtain

$$\sqrt{6x + 1} = x - 1.$$ 

2. Square both sides of the equation, and expand the results:

$$\left(\sqrt{6x + 1}\right)^2 = (x - 1)^2$$

$$6x + 1 = x^2 - 2x + 1.$$ 

3. Rewrite the equation so that all nonzero terms are on one side of the equation, and factor the nonzero side:

$$x^2 - 8x = 0$$

$$x(x - 8) = 0.$$ 

4. Using the zero-product principle, set each factor equal to zero and solve for $x$:

$$x = 0, \text{ or } x - 8 = 0 \implies x = 8.$$ 

5. Check the two values of $x$ by substituting them into the original equation,

$$\sqrt{6x + 1} + 1 = x.$$ 

Substitution of $x = 0$ into this equation gives

$$\sqrt{6 \cdot 0 + 1} + 1 = 0, \text{ or } 2 = 0,$$

which is impossible. So $x = 0$ is not a solution of the original equation. Substitution of $x = 8$ into this equation leads to

$$\sqrt{6 \cdot 8 + 1} + 1 = 8 \implies \sqrt{49} + 1 = 8 \implies 8 = 8,$$

a true statement. So $x = 8$ solves the original equation.

The solution is: $$$8$$.