Example

Solve the following equation for $x$:

$$\sqrt{6x + 1} - \sqrt{x + 1} = 4.$$

Solution: This is an equation involving two radicals as well as terms not involving a radical.

1. Isolate the two radicals by moving them to separate sides of the equation:

$$\sqrt{6x + 1} = \sqrt{x + 1} + 4.$$

2. Square both sides of the equation,

$$\left(\sqrt{6x + 1}\right)^2 = \left(\sqrt{x + 1} + 4\right)^2$$

$$6x + 1 = \left(\sqrt{x + 1}\right)^2 + 2 \cdot 4 \cdot \sqrt{x + 1} + 4^2$$

$$6x + 1 = x + 1 + 8\sqrt{x + 1} + 16,$$

which is again a radical equation, except now with only one radical term.

3. Rewrite the last equation so that the radical term is on one side of the equation and all other terms are on the other side, i.e.,

$$5x - 16 = 8\sqrt{x + 1},$$

and again square both sides of the equation to remove the remaining radical:

$$\left(5x - 16\right)^2 = \left(8\sqrt{x + 1}\right)^2$$

$$25x^2 - 160x + 256 = 64(x + 1).$$

4. Rewrite the last equation so that all nonzero terms are on one side of the equation:

$$25x^2 - 160x + 256 - 64x - 64 = 0$$

$$25x^2 - 224x + 192 = 0.$$
5. Using the quadratic formula,

\[
x = \frac{-(−224) \pm \sqrt{(−224)^2 - 4 \cdot 25 \cdot 192}}{2 \cdot 25}
\]

\[
= \frac{224 \pm \sqrt{30976}}{50}
\]

\[
= \frac{224 \pm 176}{50}
\]

so

\[
x = \frac{400}{50} = 8, \quad \text{or} \quad x = \frac{48}{50} = \frac{24}{25}.
\]

6. Check the two values of \(x\) by substituting them into the original equation,

\[
\sqrt{6x + 1} - \sqrt{x + 1} = 4.
\]

Substitution of \(x = 8\) into this equation gives

\[
\sqrt{6 \cdot 8 + 1} - \sqrt{8 + 1} = 7 - 3 = 4,
\]

which verifies that \(x = 8\) is a solution. Substitution of \(x = 24/25\) into this equation leads to

\[
\sqrt{6 \cdot \frac{24}{25} + 1} - \sqrt{\frac{24}{25} + 1} = \sqrt{\frac{169}{25}} - \sqrt{\frac{49}{25}} = \frac{13}{5} - \frac{7}{5} = \frac{6}{5} \neq 4,
\]

so \(x = 24/25\) is not a solution of the original equation.

The solution is: \{ 8 \}.