Example

Consider the functions \( f(x) = \sqrt{x - 2} \) and \( g(x) = 3x + 1 \).

1. Find and simplify \( f + g, f - g, \) and \( fg \). Then find the domain for each.

**ANSWER:**

\[
\begin{align*}
(f + g)(x) &= f(x) + g(x) = \sqrt{x - 2} + 3x + 1 \\
(f - g)(x) &= f(x) - g(x) = \sqrt{x - 2} - (3x + 1) = \sqrt{x - 2} - 3x - 1 \\
(fg)(x) &= f(x)g(x) = (3x + 1)\sqrt{x - 2}
\end{align*}
\]

As for the domains, the only thing to worry about in all three of these is the square root. Since anything we take a square root of must be positive or zero, then we need \( x - 2 \geq 0 \). So then \( x \geq 2 \), and the domain for all three functions is \([2, \infty)\).

2. Find \((f \circ g)(x), (g \circ f)(x)\) and \((f \circ g)(1)\). Then find the domain of \((f \circ g)(x)\).

**ANSWER:**

\[
\begin{align*}
(f \circ g)(x) &= f(g(x)) \\
&= \sqrt{g(x) - 2} \\
&= \sqrt{3x + 1 - 2} \\
&= \sqrt{3x - 1} \\
(g \circ f)(x) &= g(f(x)) \\
&= 3f(x) + 1 \\
&= 3\sqrt{x - 2} + 1
\end{align*}
\]

\((f \circ g)(1) = \sqrt{3 \cdot 1 - 1} = \sqrt{2}\).

As for the domain of \((f \circ g)(x)\), the only thing to worry about is the square root. Since we are taking the square root of \( 3x - 1 \), then we need \( 3x - 1 \geq 0 \), which means \( x \geq \frac{1}{3} \). So then the domain of \((f \circ g)(x)\) is \([\frac{1}{3}, \infty)\).