Example

Find the center and radius of the circle described by the given equation. Then find the relation’s domain and range. (Hint: Graphing will help to find the domain and range.)

\[(x - 1)^2 + (y - 2)^2 = 9\]

**ANSWER:**

From the formula for the equation of a circle, we immediately get \( h = 1 \), \( k = 2 \), and \( r^2 = 9 \), so \( r = 3 \). So the center is \((1, 2)\) and the radius is 3.

Since the center is \((1, 2)\) and the radius is 3, then the point on the circle that is farthest to the left is \((1 - 3, 2) = (-2, 2)\). The point on the circle that is farthest to the right is \((1 + 3, 2) = (4, 2)\). So the smallest \(x\)-value on the circle is \(-2\) and the largest is 4. This means the domain is \([-2, 4]\).

Since the center is \((1, 2)\) and the radius is 3, then the point on the circle that is farthest down is \((1, 2 - 3) = (1, -1)\). The point on the circle that is farthest up is \((1, 2 + 3) = (1, 5)\). So the smallest \(y\)-value on the circle is \(-1\) and the largest is 5. This means the range is \([-1, 5]\).

If instead we follow the hint and graph the circle, then we get

![Graph of the circle](image)

From the graph it is clear that \(-2 \leq x \leq 4\) and \(-1 \leq y \leq 5\). So then the domain is \([-2, 4]\) and the range is \([-1, 5]\).