Example

List all possible rational zeros for the given function.

\[ f(x) = x^3 - 4x^2 - 11x + 30 \]

Use synthetic division to test the possible rational zeros and find an actual zero. Then use your quotient from the synthetic division to find the remaining zeros of the polynomial function.

**ANSWER:**

\[
\begin{align*}
\text{Factors of 30} & = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30 \\
\text{Factors of 1} & = \pm 1
\end{align*}
\]

So then all of the possible rational zeros for \( f(x) \) are

\[ \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30 \]

This would be entered in WeBWorK as

\[ 1, 2, 3, 5, 6, 10, 15, 30, -1, -2, -3, -5, -6, -10, -15, -30 \]

Next we check for roots using synthetic division. When there are a lot of roots like this, it is not a good idea to blindly guess and check. One thing we could do is graph the function \( f(x) \) in a calculator and see where it looks like the roots are. Then we pick one and do synthetic division. Plotting the given \( f(x) \), it looks like there is a root at \( x = 2 \), so we check it with synthetic division.

\[
\begin{array}{cccc}
2 & 1 & -4 & -11 & 30 \\
& 2 & -4 & -30 \\
\hline
1 & -2 & -15 & 0
\end{array}
\]

This is a remainder of 0, which is what we want.
The synthetic division tells us that

\[ x^3 - 4x^2 - 11x + 30 = (x - 2)(x^2 - 2x - 15) \]

This verifies that \( x = 2 \) is one of the zeros. To find the remaining zeros of \( f(x) \), we therefore just need to find the zeros of the quotient \( x^2 - 2x - 15 \).

\[ x^2 - 2x - 15 = 0 \]
\[ (x + 3)(x - 5) = 0 \]
\[ x + 3 = 0 \text{ or } x - 5 = 0 \]

So the other two zeros are \( x = -3 \) and \( x = 5 \).

The zeros of \( f(x) \) are \( \{2, -3, 5\} \).