Example

Find an \( n \)th degree polynomial with real coefficients satisfying the given conditions.
\[ n = 3; \quad 1 \text{ and } i \text{ are zeros; } \quad f(2) = 10 \]

**ANSWER:**

Since \( n = 3 \), then the polynomial has degree 3, which means it has 3 zeros, counting multiplicity. Since \( x = i \) is a zero, then \( x = -i \) must also be a zero, and this gives us all three zeros.

The Factor Theorem tells us that since \( x = i, \ x = -i \) and \( x = 1 \) are zeros of \( f(x) \), then \( (x - i), (x + i), \) and \( (x - 1) \) are factors of \( f(x) \).

Also, \( f(x) \) has a leading coefficient which we will need to determine later. For now we will call it \( a_n \). Then we have

\[
f(x) = a_n(x - i)(x + i)(x - 1)
\]

\[
= a_n(x^2 + 1)(x - 1)
\]

\[
= a_n(x^3 - x^2 + x - 1)
\]

Now we will use the fact that \( f(2) = 10 \) to find \( a_n \). We know that \( f(x) = a_n(x^3 - x^2 + x - 1) \). So then

\[
f(2) = a_n(2^3 - 2^2 + 2 - 1)
\]

\[
= a_n(8 - 4 + 2 - 1)
\]

\[
= 5a_n
\]

Since we are given that \( f(2) = 10 \), then \( 5a_n = 10 \), which means \( a_n = 2 \). Then \( f(x) = 2(x^3 - x^2 + x - 1) \). We must expand the polynomial, so our final answer is

\[
f(x) = 2x^3 - 2x^2 + 2x - 2
\]