Example

Solve the polynomial inequality. Express the solution set in interval notation.

\[
x + 1 \geq \frac{1}{x + 2}
\]

**ANSWER:** \(((-\infty, -2) \cup [0, \infty))\)

First we rearrange the equation so that 0 is on one side. Let’s subtract \(\frac{1}{x + 2}\) from both sides. Then we have

\[
x + 1 \geq \frac{1}{x + 2}
\]

\[
\frac{x + 1}{x + 2} - \frac{1}{x + 2} \geq 0
\]

\[
\frac{x + 1 - 1}{x + 2} \geq 0
\]

\[
\frac{x}{x + 2} \geq 0
\]

This is the inequality we want to solve now.

Now let’s call the left-hand side \(f(x)\). So then \(f(x) = \frac{x}{x + 2}\). Next we need to find the values of \(x\) that make the numerator equal zero or the denominator equal zero. The numerator is just \(x\), which equals zero when \(x = 0\). The denominator is \(x + 2\), and \(x + 2 = 0\) when \(x = -2\). So now we use these values \(x = 0\) and \(x = -2\) to divide the real line into intervals:

So we have three intervals we need to look at: \((-\infty, -2), (-2, 0),\) and \((0, \infty)\). In each of these intervals we pick one test number. Any number in the interval will work but some are better to use than others (and remember, these are open intervals so the endpoints are not included). Here are the intervals along with the test numbers we will use:

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Test Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -2))</td>
<td>-3</td>
</tr>
<tr>
<td>((-2, 0))</td>
<td>-1</td>
</tr>
<tr>
<td>((0, \infty))</td>
<td>1</td>
</tr>
</tbody>
</table>

Now we plug each test number into \(f(x)\) to determine the behavior of \(f(x)\) in the intervals.

\[
f(-3) = \frac{-3}{-3 + 2} = \frac{-3}{-1} = 3,
\]
which is positive. So then \( f(x) \) is positive in the entire interval \((-\infty, -2)\) and we label our line accordingly:

\[
\begin{array}{c}
+ \\
-2 & 0
\end{array}
\]

Now we do the same thing with the other two test numbers.

\[
f(-1) = \frac{-1}{-1+2} = \frac{-1}{1} = -1 < 0, \text{ and } f(1) = \frac{1}{1+2} = \frac{1}{3} > 0
\]

Then we label our number line accordingly:

\[
\begin{array}{c}
+ & - & + \\
-2 & 0
\end{array}
\]

Our number line tells us that \( \frac{x}{x+2} > 0 \) on \((-\infty, -2) \cup (0, \infty)\). We want to know where \( \frac{x}{x+2} \geq 0 \), so we also need to know where \( \frac{x}{x+2} = 0 \). This happens when the numerator is zero, which is when \( x = 0 \).

So the solution set is the union \((-\infty, -2) \cup (0, \infty)\) together with the point \( x = 0 \). This is the union \((-\infty, -2) \cup [0, \infty)\), the final answer. Be careful! \( x = -2 \) is NOT part of the solution because if \( x = -2 \), then the denominator of \( f(x) \) will be zero, which is not allowed.

This answer would be entered into WeBWorK as

\((-\text{inf}, -2) \cup [0, \text{inf})\)

If we were to graph this solution on a number line (on a new one, not on the same number line we used earlier), it would be this:

\[
\begin{array}{c}
\circ & -2 & \bullet & 0
\end{array}
\]