Example

Find the domain of the function and express it in interval notation.

\[ f(x) = \sqrt{(x - 1)(x - 3)} \]

**ANSWER:** \((-\infty, 1] \cup [3, \infty)\)

The domain restrictions discussed so far are:
(1) Cannot divide by zero.
(2) Cannot take square roots of negative numbers.

Since there is no division here, we don’t need to worry about (1). We are taking a square root, so we need to make sure that \((x - 1)(x - 3)\) is not negative. In other words, we need to guarantee that \((x - 1)(x - 3)\) is positive or zero. This means we need to solve the inequality

\[(x - 1)(x - 3) \geq 0\]

First call the left-hand side \(f(x)\). So then \(f(x) = (x - 1)(x - 3)\). Next we need to find the zeros of \(f(x)\).

\((x - 1)(x - 3) = 0\) when \(x - 1 = 0\) or when \(x - 3 = 0\). So then \(x = 1\) and \(x = 3\) are the zeros of \(f(x)\), and we use these values to divide the real line into intervals:

\[
\begin{array}{ccc}
1 & & 3 \\
\end{array}
\]

So we have three intervals we need to look at: \((-\infty, 1)\), \((1, 3)\), and \((3, \infty)\). In each of these intervals we pick one test number. Any number in the interval will work but some are better to use than others (and remember, these are open intervals so the endpoints are not included). Here are the intervals along with the test numbers we will use:

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Test Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, 1))</td>
<td>0</td>
</tr>
<tr>
<td>((1, 3))</td>
<td>2</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>4</td>
</tr>
</tbody>
</table>

Now we plug each test number into \(f(x)\) to determine the behavior of \(f(x)\) in the intervals. \(f(0) = (0 - 1)(0 - 3) = 3\), which is positive. So then \(f(x)\) is positive in the entire interval \((-\infty, 1)\) and we label our line accordingly:

\[
\begin{array}{ccc}
+ & & \\
1 & & 3 \\
\end{array}
\]
Now we do the same thing with the other two test numbers.

\[ f(2) = (2 - 1)(2 - 3) = (1)(-1) = -1 < 0, \text{ and} \]
\[ f(4) = (4 - 1)(4 - 3) = (3)(1) = 3 > 0 \]

Then we label our number line accordingly:

\[ + \quad - \quad + \]
\[ \quad 1 \quad 3 \quad \]

Our number line tells us that \((x - 1)(x - 3) > 0\) on \((-\infty, 1) \cup (3, \infty)\). We want to know where \((x - 1)(x - 3) \geq 0\), so we also need to know where \((x - 1)(x - 3) = 0\). But we did this earlier when we found the zeros \(x = 1\) and \(x = 3\).

So the solution set is the union \((-\infty, 1) \cup (3, \infty)\) together with the points \(x = 1\) and \(x = 3\). This is the union \((-\infty, 1] \cup [3, \infty)\), the final answer. This would be entered in WeBWorK as

\((-\text{inf}, 1) \cup (3, \text{inf})\)