Example

Evaluate the expression without using a calculator.

\[ \ln e^4 = \underline{\quad} \]

**ANSWER:** \[ \ln e^4 = 4 \]

First we write \( x = \ln e^4 \). Then we convert this logarithmic equation to an exponential one by applying the general rule:

\[ x = \ln e^4 \iff e^x = e^4 \]

Remember that “\( \ln M \)” is short for “\( \log_e M \)” (no matter what \( M \) is), so \( x = \ln e^4 \) means \( x = \log_e e^4 \), and that is where the \( e^x \) in the exponential equation above comes from.

Now we solve the exponential equation \( e^x = e^4 \). Since the bases are equal, then the exponents must be equal as well and we can say right away that \( x = 4 \). Also, since we defined \( x \) to be \( x = \ln e^4 \), then we conclude that \( \ln e^4 = 4 \).