Partial Example

Solve the exponential equation. Express the solution as an exact answer in terms of natural logarithms. Then use a calculator to give an approximation of the solution, rounded to two decimal places.

\[ e^{2x} - e^x - 6 = 0 \]

**ANSWER:**

In this partial example we will only do the first few steps.

One way to do this is to treat it as a substitution problem. So first we note that \( e^{2x} = (e^x)^2 \). Then we write the equation as

\[
\begin{align*}
  e^{2x} - e^x - 6 &= 0 \\
  (e^x)^2 - e^x - 6 &= 0
\end{align*}
\]

Now we make the substitution \( u = e^x \). Then \( u^2 = (e^x)^2 \) and we have

\[
\begin{align*}
  e^{2x} - e^x - 6 &= 0 \\
  (e^x)^2 - e^x - 6 &= 0 \\
  u^2 - u - 6 &= 0
\end{align*}
\]

This can be factored:

\[
\begin{align*}
  e^{2x} - e^x - 6 &= 0 \\
  (e^x)^2 - e^x - 6 &= 0 \\
  u^2 - u - 6 &= 0 \\
  (u - 3)(u + 2) &= 0
\end{align*}
\]

This will give us two possible values for \( u \). Solving for \( u \) is not the end of the problem! Once we find \( u \), we need to back-substitute \( u = e^x \) to solve for \( x \), and then we need to throw out any invalid solutions.