Example

Solve the logarithmic equation. Separate multiple answers with a comma if necessary.

\[ \log_8 x + \log_8(x - 12) = 2 \]

**ANSWER:**

We want to solve for \( x \), and since we have \( x \) appearing in two different logs we need to combine them into one log. To do this we will use the property \( \log_b M + \log_b N = \log_b(MN) \).

\[ \log_8 x + \log_8(x - 12) = 2 \]
\[ \log_8 [x(x - 12)] = 2 \]
\[ \log_8(x^2 - 12x) = 2 \]

Next we convert the logarithmic equation to an exponential equation.

\[ \log_8 x + \log_8(x - 12) = 2 \]
\[ \log_8 [x(x - 12)] = 2 \]
\[ \log_8(x^2 - 12x) = 2 \]
\[ x^2 - 12x = 8^2 \]
\[ x^2 - 12x = 64 \]

This is a quadratic equation in \( x \) which can be factored.

\[ x^2 - 12x = 64 \]
\[ x^2 - 12x - 64 = 0 \]
\[ (x - 16)(x + 4) = 0 \]
\[ x = 16 \text{ or } x = -4 \]

So we have two solutions for \( x \). We need to check them to make sure they both work. If we plug \( x = 16 \) into the original equation, then we have:

\[ \log_8 x + \log_8(x - 12) = 2 \]
\[ \log_8 16 + \log_8(16 - 12) = 2 \]
\[ \log_8 16 + \log_8 4 = 2 \]
\[ \log_8 (16 \cdot 4) = 2 \]
\[ \log_8 64 = 2 \]
Since $\log_8 64 = 2$ is a true statement (because $8^2 = 64$), then $x = 16$ is a valid solution. But if we try $x = -4$, then:

$$\log_8 x + \log_8 (x - 12) = 2$$
$$\log_8 (-4) + \log_8 (-4 - 12) = 2$$

Right away we get $\log_8 (-4)$, which is not allowed since we can only take logs of **positive** numbers.

So the only valid solution is $[x = 16]$. 