Example

Find the range, period, phase shift, and equation of vertical asymptotes of the function.

(a)  \( y = 12 \csc (14x - 3); \)
(b)  \( y = 12 \sec (14x - 3). \)

**SOLUTION:**

(a) For the general function \( y = A \csc(\omega x - \phi), \) the range is \(( -\infty, -A] \cup [A, \infty)\), the period is \( \frac{2\pi}{\omega} \), the phase shift is \( \frac{\phi}{\omega} \) and the equation of vertical asymptotes is \( \omega x - \phi = n\pi \) where \( n \) is an integer.

In this case, \( A = 12 \), then the range is \(( -\infty, -12] \cup [12, \infty)\).

Also, \( \omega = 14 \), which means \( \frac{2\pi}{\omega} = \frac{\pi}{7} \). So then the period is \( \frac{\pi}{7} \).

In addition, \( \phi = 3 \), and so \( \frac{\phi}{\omega} = \frac{3}{14} \). This is the phase shift.

Finally, solve \( 14x - 3 = n\pi \) for \( x \), and we can get the equation of vertical asymptotes is \( x = \frac{3}{14} + \frac{n\pi}{14} \), where \( n \) is an integer.

(b) For the general function \( y = A \sec(\omega x - \phi), \) the range is \(( -\infty, -A] \cup [A, \infty)\), the period is \( \frac{\pi}{\omega} \), the phase shift is \( \frac{\phi}{\omega} \) and the equation of vertical asymptotes is \( \omega x - \phi = \frac{\pi}{2} + n\pi \) where \( n \) is an integer.

In this case, \( A = 12 \), then the range is \(( -\infty, -12] \cup [12, \infty)\).

Also, \( \omega = 14 \), which means \( \frac{2\pi}{\omega} = \frac{\pi}{7} \). So then the period is \( \frac{\pi}{7} \).

In addition, \( \phi = 3 \), and so \( \frac{\phi}{\omega} = \frac{3}{14} \). This is the phase shift.

Finally, solve \( 14x - 3 = \frac{\pi}{2} + n\pi \) for \( x \), and we can get the equation of vertical asymptotes is \( x = \frac{3}{14} + \frac{\pi}{28} + \frac{n\pi}{14} \), where \( n \) is an integer.