Example

1. Convert the polar coordinates \((5, \frac{\pi}{2})\) to rectangular coordinates.

2. Convert the rectangular coordinates \((3, \sqrt{3})\) to polar coordinates.

Let \(r > 0\) and \(0 \leq \theta < 2\pi\).

**SOLUTION:**

1. The rectangular coordinates are \(x = r \cos \theta\) and \(y = r \sin \theta\). Since \(r = 5\) and \(\theta = \frac{\pi}{2}\), then

   \[
   x = 5 \cos \frac{\pi}{2} = 5(0) = 0 \\
   y = 5 \sin \frac{\pi}{2} = 5(1) = 5 
   \]

   So then the rectangular coordinates are \((0, 5)\).

2. The polar coordinate \(r\) is \(r = \sqrt{x^2 + y^2}\). Since \(x = 3\) and \(y = \sqrt{3}\), then

   \[
   r = \sqrt{(3)^2 + (\sqrt{3})^2} \\
   = \sqrt{9 + 3} \\
   = \sqrt{12} \\
   = 2\sqrt{3}. 
   \]

   Also, \(\theta\) is the angle such that \(\tan \theta = \frac{y}{x} = \frac{\sqrt{3}}{3}\). Since \((3, \sqrt{3})\) is in Quadrant I, then the terminal side of \(\theta\) is also in Quadrant I. So then \(\theta\) is the angle in Quadrant I such that \(\tan \theta = \frac{\sqrt{3}}{3}\). This angle is \(\theta = \frac{\pi}{6}\).

   Therefore the polar coordinates are \(\left(2\sqrt{3}, \frac{\pi}{6}\right)\).