Example

Find all critical numbers of the function

\[ f(x) = \frac{2x}{(x^2 + 2)^2}. \]

Solution: Recall that the critical numbers of a function \( f(x) \) are those values of \( x \) in the domain of \( f \) for which \( f'(x) = 0 \) or \( f'(x) \) is undefined. Using the quotient rule,

\[
 f'(x) = \frac{(x^2 + 2)^2 \cdot 2 - 2(x^2 + 2) \cdot 2x \cdot 2x}{(x^2 + 2)^4} = \frac{(x^2 + 2)[2(x^2 + 2) - 8x^2]}{(x^2 + 2)^4},
\]

so

\[
 f'(x) = \frac{2(2 - 3x^2)}{(x^2 + 2)^3}. \tag{1}
\]

To determine the values of \( x \) for which \( f'(x) = 0 \), look at the values of \( x \) which make the numerator in (1) zero, i.e.,

\[ 2(2 - 3x^2) = 0, \]

or

\[ 3x^2 = 2 \quad \Rightarrow \quad x^2 = \frac{2}{3} \quad \Rightarrow \quad x = \pm \sqrt{\frac{2}{3}}. \]

To determine the values of \( x \) for which \( f'(x) \) is undefined, look at the values of \( x \) which make the denominator in (1) zero, or for which

\[ (x^2 + 2)^3 = 0. \]

But \( x^2 + 2 > 0 \) for all values of \( x \), so the denominator can never be zero.

Thus the critical numbers of \( f(x) \) are

\[ x = -\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}. \]