Example

Let \( f(x) = -x^3 + 2x^2 - x + 5 \). Describe the concavity of the graph of \( f(x) \) and find the points of inflection (if any).

Solution: First we have

\[
f'(x) = -3x^2 + 4x - 1.
\]

and

\[
f''(x) = -6x + 4.
\]

Note that \( f''(x) = 0 \) only at \( x = \frac{2}{3} \), and \( f'' \) keeps a constant sign on \(( -\infty, \frac{2}{3} )\) and on \(( \frac{2}{3}, \infty )\).

The sign of \( f'' \) on these intervals and the consequences for the graph of \( f \) are as follows:

<table>
<thead>
<tr>
<th>sign of ( f'' )</th>
<th>concave upward</th>
<th>point of inflection</th>
<th>concave downward</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ + + + + + + + + + +</td>
<td></td>
<td>( \frac{2}{3} )</td>
<td></td>
</tr>
</tbody>
</table>

so it follows that

- \( f \) is concave upward for \( x \in ( -\infty, \frac{2}{3} ) \),

- \( f \) is concave downward for \( x \in ( \frac{2}{3}, \infty ) \),

- \( f \) has a point of inflection at \( x = \frac{2}{3} \).