Example

Find the average rate of change of the function

\[ f(x) = \tan x - x \]

over the interval \([-\pi/4, \pi/4]\).

Solution: The average rate of change of a function \(f(x)\) over the interval \([a, b]\) is given by the slope of the secant line joining \((a, f(a))\) to \((b, f(b))\), or by

\[ \frac{f(b) - f(a)}{b - a}. \]

So the average rate of change of \(f(x) = \tan x - x\) over the interval \([-\pi/4, \pi/4]\) is given by

\[
\frac{f(\pi/4) - f(-\pi/4)}{\pi/4 - (-\pi/4)} = \frac{(\tan(\pi/4) - \pi/4) - (\tan(-\pi/4) - (-\pi/4))}{\pi/2}
\]

\[
= \frac{2}{\pi} \left[ \left(1 - \frac{\pi}{4}\right) - \left(-1 + \frac{\pi}{4}\right) \right]
\]

\[
= \frac{2}{\pi} \left(2 - \frac{\pi}{2}\right).
\]