Example

Evaluate: \( \lim_{x \to 0} \frac{x^2 - x}{2x} \).

Solution: The function \( f \) defined by

\[
f(x) = \frac{x^2 - x}{2x}
\]

is a rational function which is not defined at the value of \( x = 0 \) since the denominator of \( f \) is zero for that value of \( x \). Thus we cannot write

\[
\lim_{x \to 0} f(x) = f(0)
\]

in this case.

However, the problem asks if the function \( f \) is approaching some value as \( x \) approaches 0 from both sides (but \( x \) never actually gets to 0). In this situation we may rewrite \( f \) using

\[
f(x) = \frac{x^2 - x}{2x} = \frac{x(x - 1)}{2x} = \frac{(x - 1)}{2} \cdot \frac{x}{x}, \quad \text{for } x \neq 0,
\]

where

\[
\frac{x}{x} = 1
\]

provided \( x \neq 0 \). In the case of the above limit, \( x \) never actually equals 0, so it follows that

\[
\lim_{x \to 0} \frac{x^2 - x}{2x} = \lim_{x \to 0} \frac{(x - 1)}{2} \cdot \frac{x}{x}
\]

\[
= \lim_{x \to 0} \frac{(x - 1)}{2}
\]

\[
= \frac{0 - 1}{2}
\]

\[
= \frac{-1}{2}.
\]