Example

Evaluate: \( \lim_{x \to -2} \frac{x^2 + x - 2}{x + 2} \).

Solution: The function \( f \) defined by

\[
f(x) = \frac{x^2 + x - 2}{x + 2}
\]

is a rational function, which is not defined at the value of \( x = -2 \) since the denominator of \( f \) is zero for that value of \( x \). Thus we cannot write

\[
\lim_{x \to -2} f(x) = f(-2)
\]

in this case.

However, the problem asks if the function \( f \) is approaching some value as \( x \) approaches \(-2\) from both sides (but \( x \) never actually gets to \(-2\)). In this situation we may rewrite \( f \) using

\[
f(x) = \frac{x^2 + x - 2}{x + 2} = \frac{(x + 2)(x - 1)}{x + 2} = (x - 1) \cdot \frac{x + 2}{x + 2}, \quad x \neq -2,
\]

where

\[
\frac{x + 2}{x + 2} = 1
\]

provided \( x \neq -2 \). Since in the case of the limit \( x \) never actually equals \(-2\), it follows that

\[
\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2} = \lim_{x \to -2} (x - 1) \cdot \frac{x + 2}{x + 2} = \lim_{x \to -2} (x - 1) = -3.
\]