Example

Let the function \( f \) be defined by

\[
f(x) = \begin{cases} 
4 - x, & \text{if } x \leq -1, \\
x^2 + 1, & \text{if } x > -1
\end{cases}
\]

Is \( f \) continuous at \( x = -1 \)?

**Solution:** To show that \( f \) is continuous at \( x = -1 \), it suffices to show that the limit

\[
\lim_{x \to -1} f(x)
\]

exists and equals \( f(-1) = 4 - (-1) = 5 \).

Because the function is defined separately for \( x \) on either side of \(-1\), we’ll first compute the left-hand limit,

\[
\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (4 - x) = 4 - (-1) = 5.
\]

The right-hand limit is found as follows:

\[
\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (x^2 + 1) = (-1)^2 + 1 = 2.
\]

The left-hand and right-hand limits both exist but they are not the same value, so it follows that the full limit

\[
\lim_{x \to -1} f(x)
\]

does not exist.

Therefore the function \( f \) **not** continuous at the point \( x = -1 \), and in fact, the above shows that \( f \) has a **jump discontinuity** (an example of an **essential discontinuity**) at that point.