Example

For which value(s) of \( a \) is the function

\[
f(x) = \begin{cases} 
  x^2 - 1, & \text{for } x < 3 \\
  ax, & \text{for } x \geq 3
\end{cases}
\]

continuous at \( x = 3 \)?

Solution: For continuity at \( x = 3 \), we need to find \( a \) so that

\[
\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3).
\]

Taking the left-hand limit,

\[
\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2 - 1) = 3^2 - 1 = 8.
\]

The right-hand limit satisfies

\[
\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} ax = 3a.
\]

In order that the left-hand and right-hand limits as \( x \to 3 \) agree, it suffices to pick \( a \) so that

\[
8 = 3a, \quad \text{or} \quad a = \frac{8}{3}.
\]

With this choice we obtain

\[
\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3) = 8,
\]

and the function is continuous as \( x = 3 \).