Example

Let

\[ f(x) = \frac{4x^3 - 8x^2 + 5x - 8}{x - 2}. \]

Use long division to show that the curve \( y = f(x) \) approaches the curve of a quadratic function as \( x \to \pm \infty \). What is the equation of this quadratic function?

Solution: Rewrite the function \( f(x) = \frac{4x^3 - 8x^2 + 5x - 8}{x - 2} \) using long division:

\[
\begin{array}{r|rrrr}
& 4x^2 & + 5 \\
\hline
x - 2 & 4x^3 & - 8x^2 & + 5x & - 8 \\
& 4x^3 & - 8x^2 \\
\hline
& 5x & - 8 \\
& 5x & - 10 \\
\hline
& 2 
\end{array}
\]

Thus,

\[ f(x) = \frac{4x^3 - 8x^2 + 5x - 8}{x - 2} = 4x^2 + 5 + \frac{2}{x - 2}. \]

where

\[ \lim_{x \to \infty} \frac{2}{x - 2} = 0, \quad \text{and} \quad \lim_{x \to -\infty} \frac{2}{x - 2} = 0. \]

So as \( x \to \pm \infty \), the curve \( y = f(x) \) approaches

\[ y = 4x^2 + 5. \]