Example

Let $f(x) = 2x^2 + 4$. Use the alternative definition of the derivative,

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}. \tag{1}$$

to find $f'(x)$ for this function.

Solution: Using the definition of the derivative in (1),

$$f'(x) = \lim_{z \to x} \frac{(2z^2 + 4) - (2x^2 + 4)}{z - x}$$
$$= \lim_{z \to x} \frac{2(z^2 - x^2)}{z - x}$$
$$= \lim_{z \to x} \frac{2(z - x)(z + x)}{z - x}$$
$$= \lim_{z \to x} 2(z + x)$$
$$= 2(x + x)$$

so that we have

$$f'(x) = 4x.$$