Example

The graph below shows the velocity $v = f(t)$ of a particle moving along a horizontal coordinate line for the open interval $(0, 10)$:

(a) When does the particle move forward?

(b) When does the particle slow down?

(c) When is the particle’s acceleration positive?

(d) When does the particle stand still for more than an instant?

Solution:

(a) The particle is moving forward when its velocity $v(t)$ is positive, or when the graph of $v = f(t)$ is above the $t$-axis. So the particle is moving forward on the intervals $(2, 7) \cup (9, 10)$.

(b) If the particle is moving forward ($v(t) > 0$), then the particle is slowing down when the acceleration is negative (or when $v$ is decreasing, or $v'(t) < 0$, i.e., when the slopes of lines in the graph are negative). If the particle is moving backward ($v(t) < 0$), then the particle is slowing down when the acceleration is positive (or when $v$ is increasing, or $v'(t) > 0$, i.e., when the slopes of lines in the graph are negative).

Note that where the velocity curve has corners, the derivative of velocity (or acceleration) is not defined. So locations of corners should not be included in the answer.

Considering first only values of $t$ for which $v(t) > 0$, we note that $v(t)$ is decreasing (or $v'(t) < 0$) for $t \in (6, 7)$. And next considering only values of $t$ for which $v(t) < 0$, it is clear that $v(t)$ is increasing (or $v'(t) > 0$) for $t \in (0, 2)$. So the particle is slowing down on the intervals

$(0, 2) \cup (6, 7)$. 
(c) The particle’s acceleration is positive when \( v'(t) > 0 \), or the slopes of lines in the graph are positive. So the particle’s acceleration is positive on the intervals \((0, 4) \cup (9, 10)\). Note that asking where the acceleration is positive is \textit{not} the same as asking when the particle is speeding up, because “speeding up” is relative to the direction in which the particle is moving at the time.

(d) The particle stands still whenever its velocity \( v(t) \) is zero, or for \( t = 2 \) and \( t \in [7, 9] \). However, since we were asked for when the particle stands still for more than an instant, the answer is the interval \([7, 9]\). (Note that endpoints \textit{are} included in this case since the value of \( v \) is defined and equal to zero at the endpoints \( t = 7 \) and \( t = 9 \).)