Example

One of the curves below represents the graph of a function, and the other curve represents the graph of the derivative of that function. Which curve is which?

Solution: In other words, one graph is of a function $y = f(t)$, and the other is the graph of $y = f'(t)$.

- **Case #1:** First consider the possibility that the solid red curve is the graph of $y = f(t)$. If that is the case, then the curve of the corresponding derivative function, $y = f'(t)$, must satisfy the following properties:
  - The graph of $y = f'(t)$ must go through the point $(0, 0)$, since the solid red curve has a horizontal tangent (i.e., slope = 0) at $t = 0$.
  - The graph of $y = f'(t)$ must take on positive values (be above the $t$-axis) for $t < 0$, since the solid red curve is increasing (has positive slope) for $t < 0$.
  - The graph of $y = f'(t)$ must take on negative values (be below the $t$-axis) for $t > 0$, since the solid red curve is decreasing (has negative slope) for $t > 0$.

Although the dashed blue curve does go through $(0, 0)$, the rest of the description of $y = f'(t)$ does not match the dashed blue curve. It follows that red curve cannot be the position curve, $y = f(t)$.

- **Case #2:** To verify that it is the dashed blue curve which is the graph of $y = f(t)$, consider what the curve of $y = f'(t)$ must look like in this case. If we let $t_-$ and $t_+$ denote the $t$-values where the dashed blue curve is at its minimum point and its maximum point, respectively, then:
  - The curve of $y = f'(t)$ must cross the $t$-axis at points $t = t_-$ and $t = t_+$, since the dashed blue curve has horizontal tangents at those points.
  - The curve of $y = f'(t)$ must be above the $t$-axis on the interval $(t_-, t_+)$ because the dashed blue curve is increasing (i.e., with positive slope) on that interval.
  - The curve of $y = f'(t)$ must be below the $t$-axis for $t < t_-$ and for $t > t_+$ because the dashed blue curve is decreasing (i.e., with negative slope) for those values of $t$. 

This description matches the solid red curve. It follows that the dashed blue curve is the graph of the function \((y = f(t))\) and the solid red curve is the graph of the derivative of the function \((y = f'(t))\).