Example

Suppose that the differentiable function $g(x)$ satisfies
\[ xg(x) - x + 1 = g^3(x), \]
and that $g(0) = 1$. Find $g'(0)$.

Solution: Differentiating implicitly with respect to $x$, we have
\[ \frac{d}{dx} (xg(x) - x + 1) = \frac{d}{dx} (g^3(x)), \]
or
\[ xg'(x) + g(x) - 1 = 3g^2(x) \cdot g'(x). \]
Solving for $g'(x)$, we first collect together only the terms involving $g'(x)$ on the left-hand side of the equation,
\[ xg'(x) - 3g^2(x) \cdot g'(x) = -g(x) + 1, \]
or
\[ (x - 3g^2(x))g'(x) = -g(x) + 1, \]
which gives
\[ g'(x) = \frac{-g(x) + 1}{x - 3g^2(x)}. \]

Now, to evaluate $g'(0)$ we use the fact that $g(0) = 1$ to obtain
\[ g'(0) = \frac{-g(0) + 1}{0 - 3g^2(0)} = \frac{-1 + 1}{0 - 3 \cdot 1^2} = 0. \]