Example

Find all critical numbers of the function

\[ f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}. \]

Solution: Recall that the critical numbers of a function \( f(x) \) are those values of \( x \) in the domain of \( f \) for which \( f'(x) = 0 \) or \( f'(x) \) is undefined. Since

\[ f(x) = x^{1/2} + x^{-1/2}, \]

it follows that

\[ f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} \]

\[ = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}. \]

Finding a common denominator, we have

\[ f'(x) = \frac{x - 1}{2x^{3/2}}. \quad (1) \]

To determine the values of \( x \) for which \( f'(x) = 0 \), we can look at the values of \( x \) which make the numerator in (1) zero, i.e.,

\[ x - 1 = 0 \quad \text{or} \quad x = 1. \]

To determine the values of \( x \) for which \( f'(x) \) is undefined, it suffices to look at the values of \( x \) which make the denominator in (1) zero, or for which

\[ 2x^{3/2} = 0, \quad \text{or} \quad x = 0. \]

Note however that even though \( f' \) is undefined at \( x = 0 \), the original function \( f \) is not defined at that point either, so \( x = 0 \) is not a critical number for \( f \).

So the function \( f \) has only one critical number, at

\[ x = 1. \]