Example

Let
\[ f(x) = 1 - (x - 2)^{2/3}. \]
Find the intervals where \( f \) is increasing and/or decreasing and the \( x \)-values where \( f \) has local maxima or minima.

**Solution:** The domain of \( f \) is \((-\infty, \infty)\), which is divided into subintervals by the critical numbers of \( f \). To determine the critical numbers, we compute
\[
f'(x) = -\frac{2}{3}(x - 2)^{-1/3} = -\frac{2}{3(x - 2)^{1/3}},
\]
and note that \( f'(x) \) can never be zero, but that \( f' \) is undefined where the denominator of \( f' \) in (1) is zero, i.e., at
\[
3(x - 2)^{1/3} = 0 \implies x = 2.
\]
So \( f \) has one critical number, at \( x = 2 \).

This critical number divides the real line into the open intervals \((-\infty, 2)\) and \((2, \infty)\). To determine whether \( f \) is increasing or decreasing on these intervals we make a sign chart with these intervals along the first column, and denominator of \( f' \) along the first row:

<table>
<thead>
<tr>
<th>interval</th>
<th>(3(x - 2)^{1/3})</th>
<th>(f'(x) = -2/\left(3(x - 2)^{1/3}\right))</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, 2))</td>
<td>-</td>
<td>+</td>
<td>increasing</td>
</tr>
<tr>
<td>((2, \infty))</td>
<td>+</td>
<td>-</td>
<td>decreasing</td>
</tr>
</tbody>
</table>

Thus \( f \) is increasing on \((-\infty, 2)\) and decreasing on \((2, \infty)\).

The only candidate for a local maximum or minimum of \( f \) is the critical number \( x = 2 \). Because \( f \) changes from increasing to decreasing at the point \( x = 2 \), it follows that \( f \) has a **local maximum** at \((2, f(2)) = (2, 1)\).