Example

Find the most general antiderivative of

\[ f(x) = 4x^2 - \frac{1}{x^3} + \sin 2x. \]

Solution: Note that \( f(x) \) can be rewritten as

\[ f(x) = 4x^2 - x^{-3} + \sin 2x. \]

Using the linearity of antiderivatives, the fact that an antiderivative of \( x^n \) is given by \( \frac{x^{n+1}}{n+1} \) (when \( n \neq -1 \)), and that an antiderivative of \( \sin kx \) is \( -\frac{1}{k} \cos kx \) (for \( k \neq 0 \)), it follows that the general antiderivative of \( f(x) \) is given by

\[ 4 \cdot \frac{x^3}{3} - \frac{x^{-2}}{-2} - \frac{1}{2} \cos 2x + C = \frac{4x^3}{3} + \frac{1}{2x^2} - \frac{1}{2} \cos 2x + C. \]

Note: The answer can always be checked by differentiating the answer given above and confirming that the integrand \( f(x) \) is recovered. That is,

\[
\frac{d}{dx} \left( \frac{4x^3}{3} + \frac{1}{2x^2} - \frac{1}{2} \cos 2x + C \right) = \frac{4}{3} \cdot 3x^2 + (-2)\frac{1}{2}x^{-3} - \frac{1}{2} (-\sin 2x) \cdot 2
\]

\[ = 4x^2 - \frac{1}{x^3} + \sin 2x. \]