Example

Find $F'(x)$ if

$$F(x) = \int_5^{x^3} t^{2/3} \, dt.$$ 

**Solution:** We cannot directly appeal to the result that, for $f$ continuous on an interval containing $a$ and $x$, then

$$\frac{d}{dx} \int_a^x f(t) \, dt = f(x),$$

because the upper limit of the definite integral is not $x$ alone. However, we can still find $F'(x)$ by making use of the Chain Rule.

To this end let

$$F(u) = \int_5^u t^{2/3} \, dt,$$

where

$$u = x^3.$$ 

Then,

$$\frac{dF}{dx} = \left. \frac{dF}{du} \right|_{u=x^3} \frac{du}{dx}$$

$$= u^{2/3} \bigg|_{u=x^3} \cdot 3x^2$$

$$= (x^3)^{2/3} \cdot 3x^2$$

$$= x^2 \cdot 3x^2$$

$$= 3x^4.$$