Example

Evaluate
\[ \frac{d}{dx} \left( \int_{5x}^{x^2} \frac{1}{t+1} \, dt \right) \]

Solution: First we note that
\[
\int_{5x}^{x^2} \frac{1}{t+1} \, dt = \int_{5x}^{0} \frac{1}{t+1} \, dt + \int_{0}^{x^2} \frac{1}{t+1} \, dt
\]
where we could have used any constant in place of the zero. Then
\[
\int_{5x}^{x^2} \frac{1}{t+1} \, dt = -\int_{0}^{5x} \frac{1}{t+1} \, dt + \int_{0}^{x^2} \frac{1}{t+1} \, dt,
\]
so that now we can use the Chain Rule along with the result that, for \( f \) continuous on an interval including \( x \) and \( a \),
\[ \frac{d}{dx} \int_{a}^{x} f(t) \, dt = f(x). \]
That is,
\[ \frac{d}{dx} \left( -\int_{0}^{5x} \frac{1}{t+1} \, dt \right) = -\frac{1}{(5x)+1} \cdot \frac{d}{dx} (5x) = -\frac{5}{5x+1}, \]
and
\[ \frac{d}{dx} \left( \int_{0}^{x^2} \frac{1}{t+1} \, dt \right) = \frac{1}{(x^2)+1} \cdot \frac{d}{dx} (x^2) = \frac{2x}{x^2+1}. \]
It follows then that
\[ \frac{d}{dx} \left( \int_{5x}^{x^2} \frac{1}{t+1} \, dt \right) = -\frac{5}{5x+1} + \frac{2x}{x^2+1}. \]