Example

Evaluate the indefinite integral

$$\int x\sqrt{x + 8} \, dx.$$ 

Solution: To simplify the problem, let $u = u(x)$ denote the function of $x$ under the cube root, i.e.,

$$u = x + 8,$$

so that the differential $du$ satisfies

$$du = dx. \tag{1}$$

and

$$x = u - 8.$$ 

Thus the simplification of the original integral occurs when substitutions are made using the above quantities. That is,

$$\int x\sqrt{x + 8} \, dx = \int (u - 8)\sqrt{u} \, du = \int u\sqrt{u} \, du - 8 \int \sqrt{u} \, du = \int u^{4/3} \, du - 8 \int u^{1/3} \, du,$$

an indefinite integral in the new variable of integration $u$, and one for which the solution is easily found. Thus

$$\int x\sqrt{x + 8} \, dx = \int u^{4/3} \, du - 8 \int u^{1/3} \, du = \frac{3}{7} u^{7/3} - 8 \cdot \frac{3}{4} u^{4/3} + C = \frac{3}{7} u^{7/3} - 6 u^{4/3} + C.$$

Now replacing $u$ by $x + 8$ in the solution given above, it follows that

$$\int x\sqrt{x + 8} \, dx = \frac{3}{7} (x + 8)^{7/3} - 6 (x + 8)^{4/3} + C,$$

so that the final result is entirely in terms of $x$, the original variable of integration.