Example

Let $f(x) = 2 - \frac{1}{x}$.

- Find the equation of the tangent line to the curve $y = f(x)$ at the point $(-1, f(-1))$.
- Find the equation of the normal line to the curve $y = f(x)$ at the point $(-1, f(-1))$.

Solution: We first rewrite $f$ using negative powers on $x$, i.e.,

$$f(x) = 2 - x^{-1}.$$  

Using the rules of differentiation,

$$f'(x) = (2)' - (x^{-1})'$$

$$= 0 - (-1)x^{-2}$$

$$= \frac{1}{x^2},$$

so the slope of the tangent to the curve at the point $x = -1$ is

$$f'(-1) = \frac{1}{(-1)^2} = 1.$$

- **Tangent line:** The equation of the line with slope $m = 1$ and going through the point $(-1, f(-1)) = (-1, 2 - \frac{1}{(-1)}) = (-1, 3)$ is found using the point-slope formula. It follows that the equation of the tangent line to the curve at the point $(-1, 3)$ is given by

$$y - 3 = 1 \cdot (x - (-1)), \quad \text{or} \quad y = 3 + (x + 1) \quad \text{or} \quad y = x + 4.$$

- **Normal line:** The line normal to the curve at $x = -1$ is perpendicular to the tangent line to the curve at $x = -1$. Since the slope of the tangent line is 1, the slope of the normal line is

$$m = -\frac{1}{1} = -1$$

(the negative reciprocal of the slope of the tangent line).

Again using the point-slope formula, the equation of the line with slope $m = -1$ and going through the point $(-1, 3)$ is found using the point-slope formula to be

$$y - 3 = -1 \cdot (x - (-1)), \quad \text{or} \quad y = 3 - (x + 1) \quad \text{or} \quad y = -x + 2.$$