Example

A particle moves according to the following law of motion

\[ s(t) = t + \frac{4}{t}, \quad t > 0, \]

where \( s \) is measured in meters and \( t \) in seconds.

(a) What is the average velocity of the particle over the time interval \([1, 3]\)?

(b) What is the (instantaneous) velocity of the particle at time \( t = 3 \) sec?

(c) What is the acceleration of the particle at time \( t = 3 \) sec?

(d) When is the particle moving in the positive direction, when is it moving in the negative direction, and when is it at rest?

Solution:

(a) The average velocity over the interval \([1, 3]\) is given by

\[ \frac{s(3) - s(1)}{3 - 1} = \frac{(3 + \frac{4}{3}) - (1 + \frac{4}{1})}{2} \text{ m/sec.} \]

(b) The instantaneous velocity is \( s'(t) \), where

\[ s'(t) = 1 - \frac{4}{t^2} \text{ m/sec, \ so \ } s'(3) = 1 - \frac{4}{3^2} = \frac{9 - 4}{9} = \frac{5}{9} \text{ m/sec.} \]

(c) Acceleration is given by \( s''(t) \text{ m/sec}^2 \), where

\[ s''(t) = (-2) \left( -\frac{4}{t^3} \right) = \frac{8}{t^3} \text{ m/sec}^2, \text{ \ so \ } s''(3) = \frac{8}{3^3} = \frac{8}{27} \text{ m/sec}^2. \]

(d) The particle is moving in the positive direction when \( s'(t) > 0 \), so we will first determine where \( s'(t) = 0 \). Combining the terms in \( s'(t) \),

\[ s'(t) = \frac{t^2 - 4}{t^2} = \frac{1}{t^2} (t^2 - 4) = \frac{1}{t^2} (t - 2)(t + 2), \]

and since \( 1/t^2 > 0 \) for all \( t > 0 \), we have that

\[ s'(t) = 0, \text{ \ when \ } (t - 2)(t + 2) = 0, \text{ \ or \ } t = 2, -2 \text{ sec.} \]

However, \( t = -2 \) is not within the given time interval \( t \in (0, \infty) \), so the only value of \( t \) in this interval for which \( s'(t) = 0 \) is \( t = 2 \), which means that \( s'(t) \neq 0 \) for \( t \in (0, 2) \) and \( t \in (2, \infty) \). To determine where \( s'(t) \) is positive and negative, we can make a sign chart for the factors in \( s'(t) \) when restricted to these two intervals:
The signs ’+’ or ’−’ in the table can determined by checking the value of the given factor at a single point in the interval. The conclusions to be drawn from the table are as follows:

- The particle is moving in the positive direction when the velocity satisfies $s'(t) > 0$, or when $t \in (2, \infty)$.
- It is moving in the negative direction when the velocity satisfies $s'(t) < 0$, or when $t \in (0, 2)$.
- It is at rest when $s'(t) = 0$, or at $t = 2$ sec.