Example

A ball is thrown upward from a height of 32 meters on the planet Juno, with the height of the ball given by the function

\[ y(t) = -4t^2 + 8t + 32 \text{ m}, \]

where the time \( t \geq 0 \) is measured in seconds and \( t = 0 \) corresponds to the moment the ball is thrown.

(a) When does the ball reach its maximum height?

(b) What is the maximum height of the ball?

(c) When does the ball hit the ground?

Solution:

(a) The ball reaches its maximum height when the velocity is zero, or when

\[ y'(t) = -8t + 8 = 0, \quad \text{or at } t = 1 \text{ sec.} \]

(b) The maximum height of the ball is the height \( y(t) \) at \( t = 1 \) sec, or

\[ y(1) = -4 \cdot 1^2 + 8 \cdot 1 + 32 = 36 \text{ m}. \]

(c) The ball hits the ground at the first value of \( t > 0 \) for which \( y(t) = 0 \), i.e., when

\[ y(t) = -4t^2 + 8t + 32 = -4(t^2 - 2t - 8) = -4(t - 4)(t + 2) = 0, \]

so it must be that the ball hits the ground at \( t = 4 \) sec.