Example

Let

\[ f(x) = 2 - 5x^3 \]

Determine the domain of \( f \) and its critical points. Then find the intervals where \( f'(x) > 0 \) and \( f'(x) < 0 \), and where \( f \) has local (relative) extrema.

Solution: The domain of \( f \) is \((-\infty, \infty)\), since it is a polynomial. To determine the critical points of \( f \), we compute

\[ f'(x) = -15x^2 \]

and note that \( f'(x) \) is never undefined, and that it is zero only at \( x = 0 \). Thus the number 0 is the only critical point.

The critical point divides the domain of \( f \) into two open intervals \((-\infty, 0)\) and \((0, \infty)\).

To determine whether \( f \) is increasing or decreasing on these intervals we make a sign chart with these intervals along the first column and factors of \( f' \) along the first row:

<table>
<thead>
<tr>
<th>interval</th>
<th>( x^2 )</th>
<th>( f'(x) = -15x^2 )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, 0))</td>
<td>+</td>
<td>-</td>
<td>decreasing</td>
</tr>
<tr>
<td>((0, \infty))</td>
<td>+</td>
<td>-</td>
<td>decreasing</td>
</tr>
</tbody>
</table>

Thus \( f' > 0 \) nowhere and \( f' < 0 \) on \((-\infty, 0)\) and \((0, \infty)\).

The critical point \( x = 0 \) is the only candidate local/relative maximum/minimum of \( f \):

- Because \( f \) decreases immediately before and after the point \( x = 0 \), it follows that \( f \) has no local/relative extrema at \( x = 0 \).