Example

Let \( f(x) = \frac{1}{x + 1} \). Describe the concavity of the graph of \( f(x) \) and find the points of inflection (if any).

**Solution:** First we have

\[
f'(x) = -\frac{1}{(x + 1)^2}.
\]

and

\[
f''(x) = \frac{2}{(x + 1)^3}.
\]

Note that \( f''(x) \) does not exist at \( x = -1 \), and \( f'' \) keeps a constant sign on \((-\infty, -1)\) and on \((1, \infty)\). The sign of \( f'' \) on these intervals and the consequences for the graph of \( f \) are as follows:

- – – – – – – – – undefined – – – – – – – –
- graph of \( f \): concave down \( -1 \) concave up

so it follows that

- \( f \) is concave up for \( x \in (-1, \infty) \),
- \( f \) is concave down for \( x \in (-\infty, -1) \).

Since \( x = -1 \) is not in the domain of \( f \),

- \( f \) has no points of inflection.