Example

Let \( f(x) = x - x^{1/3} \). Describe the concavity of the graph of \( f(x) \) and find the points of inflection (if any).

Solution: First we have
\[
f'(x) = 1 - \frac{1}{3x^{2/3}}.
\]
and
\[
f''(x) = \frac{2}{9x^{5/3}}.
\]
Note that \( f''(x) \) does not exist at \( x = 0 \), and \( f'' \) keeps a constant sign on \( (-\infty, 0) \) and on \( (0, \infty) \). The sign of \( f'' \) on these intervals and the consequences for the graph of \( f \) are as follows:

<table>
<thead>
<tr>
<th>sign of ( f'' ):</th>
<th>$-$  $-$  $-$  $-$  $-$  $-$  $-$  undefined  $+$  $+$  $+$  $+$  $+$  $+$  $+$  $+$  $+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>graph of ( f ):</td>
<td>concave down  0  concave up</td>
</tr>
</tbody>
</table>

so it follows that

- \( f \) is concave up for \( x \in (0, \infty) \),
- \( f \) is concave down for \( x \in (-\infty, 0) \).

Since \( f \) is continuous at 0,

- \( f \) has a point of inflection at \( x = 0 \).