Example

Let 

\[ f(x) = |x - 2|. \]

Find the largest region where \( f \) is increasing and/or decreasing.

Solution: First remove the absolute value sign, which splits the function into parts:

\[
f(x) = \begin{cases} 
  x - 2, & x > 2 \\
  0, & x = 2 \\
  -x + 2, & x < 2 
\end{cases}
\]

For \( x > 2 \): The derivative of \( x - 2 \) is 1. So \( f' > 0 \) and thus \( f \) is increasing on \((2, \infty)\).

For \( x < 2 \): The derivative of \(-x + 2\) is -1. So \( f' < 0 \) and thus \( f \) is decreasing on \((-\infty, 2)\).

Since \( f \) is defined and continuous at \( x = 2 \), the endpoint should be included in both intervals. So we can conclude that \( f \) is increasing on \([2, \infty)\) and decreasing on \((-\infty, 2]\).