Example

Let \( f(x) = -\sqrt{3}x - \cos 2x, 0 \leq x \leq \pi \). Find the largest region where \( f \) is increasing and/or decreasing.

Solution: In this case \( f'(x) = -\sqrt{3} + 2 \sin 2x \). Setting \( f'(x) = 0 \), we have

\[
-\sqrt{3} + 2 \sin 2x = 0 \Rightarrow \sin 2x = \frac{\sqrt{3}}{2}
\]

\[
\Rightarrow 2x = \frac{\pi}{3}, \frac{2\pi}{3}, \text{ when } 0 \leq x \leq \pi,
\]

\[
\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}.
\]

It follows that on the intervals \([0, \frac{\pi}{6}],[\frac{\pi}{6}, \frac{\pi}{3}],[\frac{\pi}{3}, \pi]\), the derivative \( f' \) keeps a constant sign. The sign of \( f' \) and the behavior of \( f \) are recorded below.

| sign of \( f' \): | - - - - 0 ++++ + 0 - - - - - - - - - - - - - - - - - |
| behavior of \( f \): | decreases | increases | decreases |

Since \( f \) is continuous throughout, it follows that \( f \) is increasing on \([\frac{\pi}{6}, \frac{\pi}{3}]\) and decreasing on \([0, \frac{\pi}{6}] \cup [\frac{\pi}{3}, \pi]\).