Example

A particular curve is represented parametrically by

\[ x = 2 \cos(8t) \quad y = -5 \sin(8t) \quad t \in [0, \frac{\pi}{8}]. \]

1. Find the corresponding Cartesian equation for this curve. (Write the equation of the full curve, even if only part of the curve is given by the parametrization.)

2. As \( t \) increases on \([0, \frac{\pi}{8}]\), in which direction is \((x(t), y(t))\) moving?

3. Give the largest and smallest values of \( y \) taken by this curve.

1. **Find the Cartesian equation for the curve.**

   From the parametric equations, it follows that
   \[ \frac{x}{2} = \cos(8t) \quad \text{and} \quad \frac{y}{-5} = \sin(8t), \]
   thus using the Pythagorean identity \( \cos^2(8t) + \sin^2(8t) = 1 \), the Cartesian equation for the whole curve is given by
   \[ \left( \frac{x}{2} \right)^2 + \left( \frac{y}{-5} \right)^2 = 1. \]
   The equation may be written equivalently as
   \[ \frac{x^2}{4} + \frac{y^2}{25} = 1, \]
   whose graph is an ellipse with \(-2 \leq x \leq 2\) and \(-5 \leq y \leq 5\).

2. **Determine the direction.**

   To determine the portion of the curve given by the parametric equations for \( t \in [0, \frac{\pi}{8}] \), we note the following.

   * \( t = 0 \) corresponds to the point on the curve \((2 \cos(0), -5 \sin(0)) = (2, 0)\).
   * As \( t \) increases from 0 to \( \frac{\pi}{16} \), the value of \( \cos(8t) \) decreases from 1 to 0, while the value of \( \sin(8t) \) increases from 0 to 1 (so \( y \) becomes more negative).
     \( t = \frac{\pi}{16} \) corresponds to the point on the curve \((2 \cos(\pi/2), -5 \sin(\pi/2)) = (0, -5)\).
   * As \( t \) continues to increase from \( t = \frac{\pi}{16} \) to \( t = \frac{\pi}{8} \) the value of \( \cos(8t) \) decreases from 0 to -1, while the value of \( \sin(8t) \) decreases from 1 to 0.
     \( t = \frac{\pi}{8} \) corresponds to the point on the curve \((2 \cos(\pi), -5 \sin(\pi)) = (-2, 0)\).
We conclude that the parametric equations trace out the curve \textbf{clockwise} as \( t \) increases. The equations above are a parameterization for the \textbf{lower half of the ellipse}.

3. \textbf{Find the largest and smallest values of } \( y \).
   For \( t \) in the interval \([0, \frac{\pi}{8}]\), \( \sin(8t) \) attains all values between 0 and 1. It follows that for \( y = -5 \sin(8t) \),
   \[-5 \leq y \leq 0.\]