Example

Find a parametrization of the line segment starting at the point \((x, y) = (2, 1)\) and ending at the point \((x, y) = (-3, 6)\) using affine functions \(x = x(t)\) and \(y = y(t)\), for \(t \in [0, 1]\).

Solution: First find \(x(t)\) as an affine function of the form \(x(t) = at + b\), where constants \(a\) and \(b\) are selected so that
\[
x(0) = a \cdot 0 + b = 2, \quad x(1) = a \cdot 1 + b = -3.
\]
So it must be that \(b = 2\) and \(a = -5\) and the parametrization for \(x(t)\) is
\[
x(t) = -5t + 2.
\]
Similarly, we find \(y(t)\) to be of affine form, \(y(t) = ct + d\), where \(c\) and \(d\) are selected so that
\[
y(0) = c \cdot 0 + d = 1, \quad y(1) = c \cdot 1 + d = 6.
\]
It follows that \(d = 1\) and \(c = 5\), so the parametrization of \(y(t)\) is given by
\[
y = 5t + 1.
\]
Therefore the parameterization is
\[
x(t) = -5t + 2 \quad y(t) = 5t + 1, \quad t \in [0, 1].
\]