Example

The polar equation $r = 5 \cos \theta$ describes a circle. Rewrite this equation in Cartesian coordinates. Use your result to determine the center and radius of the circle.

To facilitate substitution, first multiply both sides of the equation by $r$ to obtain

$$r^2 = 5r \cos \theta.$$  

Using the relations $x^2 + y^2 = r^2$ and $x = r \cos \theta$, the corresponding Cartesian equation is

$$x^2 + y^2 = 5x,$$

or

$$x^2 - 5x + y^2 = 0,$$

which is the equation of a circle. To determine the center and radius, put the equation of the circle in standard form by completing the square in $x$.

$$x^2 - 5x + y^2 = 0$$

$$x^2 - 5x + \left( -\frac{5}{2} \right)^2 + y^2 - \left( -\frac{5}{2} \right)^2 = 0$$

$$x^2 - 5x + \frac{25}{4} + y^2 - \frac{25}{4} = 0$$

$$\left( x - \frac{5}{2} \right)^2 + y^2 = \frac{25}{4}$$

$$\left( x - \frac{5}{2} \right)^2 + (y - 0)^2 = \left( \frac{5}{2} \right)^2.$$  

Therefore the circle has center $\left( \frac{5}{2}, 0 \right)$ and radius $\frac{5}{2}$.  