Example

Sketch the graph of the curve

\[ r^2 = 2 \cos \theta. \]

Note that if \((r, \theta)\) lies on the graph of \(r^2 = 2 \cos \theta\), then

- \((r, -\theta)\) lies on the graph since \(2 \cos(-\theta) = 2 \cos \theta = r^2\), so the curve is symmetric about the x-axis.
- \((-r, \theta)\) lies on the graph since \((-r)^2 = r^2\), so the curve is symmetric about the origin.
- from the two previous symmetries it follows that \((-r, -\theta)\) lies on the graph since \(2 \cos(-\theta) = 2 \cos \theta = r^2 = (-r)^2\), so the curve is symmetric about the y-axis.

We plot the curve by hand by constructing a table of values \((r, \theta)\) evaluating the polar curve at increasing values of \(\theta\) starting at \(\theta = 0\). The approximate values of \(r\) in the table are obtained using the (approximate) values of the cosine function found in the handout Sine and Cosine for Standard Angles (linked to some of the problems in this WeBWorK section).

Since \(r^2 = 2 \cos \theta\) implies \(\cos \theta\) must be positive, the entire graph can be traced out for \(\theta \in [0, \pi/2] \cup [3\pi/2, 2\pi]\). Due to symmetry, we first consider \(\theta \in [0, \pi/2]\).

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>0</th>
<th>(\pi / 6)</th>
<th>(\pi / 4)</th>
<th>(\pi / 3)</th>
<th>(\pi / 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = \pm \sqrt{2 \cos \theta})</td>
<td>(-\sqrt{2}, \sqrt{2})</td>
<td>(-\sqrt{3}, \sqrt{3})</td>
<td>(\sqrt{-\sqrt{2}}, \sqrt{\sqrt{2}})</td>
<td>(-1, 1)</td>
<td>0</td>
</tr>
<tr>
<td>approx. (r)</td>
<td>(-1.4, 1.4)</td>
<td>(-1.3, 1.3)</td>
<td>(-1.2, 1.2)</td>
<td>(-1, 1)</td>
<td>0</td>
</tr>
</tbody>
</table>

Using symmetry, we obtain the complete graph for \(\theta \in [0, \pi/2] \cup [3\pi/2, 2\pi]\).

Using the page of graphs, we label this curve as 2H (lemniscate horizontal). Although this information is not needed for every problem, we note that the point \(P\) corresponding to the angle \(\theta = \pi / 2\) has polar coordinates \((r, \theta) = (0, \pi / 2)\), and the point \(Q\) corresponding to the angle \(\theta = 0\) has polar coordinates \((r, \theta) = (\sqrt{2}, 0)\).
Figure 1: Graph of $r^2 = 2 \cos \theta$ for $0 \leq \theta \leq \pi/2$.

Figure 2: Graph of $r^2 = 2 \cos \theta$ for $\theta \in [0, \pi/2] \cup [3\pi/2, 2\pi]$. 