Example

Consider the following sequence:

\[ \frac{-2}{1}, \frac{4}{2}, \frac{-8}{3}, \frac{16}{4}, \ldots \]

Write a formula for the nth term \( a_n \) in this sequence.

Solution: The general terms of a sequence are expressed as

\[ a_1, a_2, a_3, a_4, \ldots, \]

so it must be in this case that

\[ a_1 = \frac{-2}{1} = -2^{1}, \quad a_2 = \frac{4}{2} = 2^{2}, \]
\[ a_3 = \frac{-8}{3} = -2^{3}, \quad a_4 = \frac{16}{4} = 2^{4}, \]

and so on. Thus the nth term may be written \( a_n = \pm \frac{2^n}{n} \), where it remains to determine the precise sign of \( a_n \) as a function of \( n \).

The usual way to represent an alternating sign in a sequence is via the use of

\[ (-1)^n = \begin{cases} -1, & n \text{ odd,} \\ 1, & n \text{ even,} \end{cases} \]

or

\[ (-1)^{n+1} = \begin{cases} 1, & n \text{ odd,} \\ -1, & n \text{ even,} \end{cases} \]

(equivalently \((-1)^{n-1}\) may be used in place of the latter). In this particular example, the sequence is negative for odd values of \( n \) and positive for even values of \( n \), so we can write the nth term as

\[ a_n = (-1)^n \frac{2^n}{n}. \]