Example

Consider the sequence \{a_n\} where the nth term is given by

\[ a_n = \sqrt[10]{10}. \]

Determine if the sequence converges or diverges. If the sequence converges, find its limit.

Solution: The \( n \)th term can be written as

\[ a_n = \sqrt[10]{10} = (10)^{1/n} = \exp \left( \ln (10)^{1/n} \right) = \exp \left( \frac{1}{n} \ln 10 \right). \]

Then

\[ \lim_{n \to \infty} a_n = \lim_{n \to \infty} \exp \left( \frac{1}{n} \ln 10 \right). \]

But

\[ \lim_{n \to \infty} \frac{1}{n} \ln 10 = \lim_{n \to \infty} \frac{\ln 10}{n} = 0, \]

and because the function \( f(x) = e^x \) is a continuous function, we may take this limit into the exponent, i.e.,

\[ \lim_{n \to \infty} \exp \left( \frac{1}{n} \ln 10 \right) = \exp (0) = 1. \]

Therefore,

\[ \lim_{n \to \infty} \sqrt[10]{10} = 1, \]

and the sequence \{a_n\} converges to the limit 1.