Example

Consider the sequence \( \{a_n\} \) where the \( n \)th term is given by

\[
a_n = \sqrt[3]{3n^2}.
\]

Determine if the sequence converges or diverges. If the sequence converges, find its limit.

**Solution:** The \( n \)th term can be written as

\[
a_n = \sqrt[3]{3n^2} = (3n^2)^{1/n} = \exp \left( \ln (3n^2)^{1/n} \right) = \exp \left( \frac{1}{n} \ln 3n^2 \right).
\]

Then

\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \exp \left( \frac{1}{n} \ln 3n^2 \right) = \exp \left( \lim_{n \to \infty} \frac{\ln 3n^2}{n} \right),
\]

recalling that we can take the limit into the exponent since \( e^x \) is a continuous function. Using l'Hopital’s rule

\[
\lim_{n \to \infty} \frac{\ln 3n^2}{n} = \lim_{n \to \infty} \frac{6n/(3n^2)}{1} = \lim_{n \to \infty} \frac{2}{n} = 0,
\]

therefore

\[
\lim_{n \to \infty} a_n = e^0 = 1,
\]

so the infinite sequence converges to the limit 1.