Example

Consider the sequence \( \{a_n\} \) where the \( n \)th term is given by

\[
a_n = \frac{1}{n} \ln 3n^2.
\]

Determine if the sequence converges or diverges. If the sequence converges, find its limit.

Solution:  Note that

\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \left( \frac{1}{n} \ln 3n^2 \right)
\]

\[
= \lim_{n \to \infty} \frac{\ln 3n^2}{n},
\]

so using l’Hopital’s rule,

\[
\lim_{n \to \infty} \frac{\ln 3n^2}{n} = \lim_{n \to \infty} \frac{6n/(3n^2)}{1} = \lim_{n \to \infty} \frac{2}{n} = 0.
\]

Therefore,

\[
\lim_{n \to \infty} a_n = 0,
\]

so the infinite sequence converges to the limit 0.