Example

Use the ratio test to determine if the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n^3 e^{2n}}{n!5^n}. \]

Find the limit of the ratio, \( L = \lim_{n \to \infty} \frac{a_{n+1}}{a_n} \).

Computing the ratio

\[
\frac{a_{n+1}}{a_n} = \frac{(n+1)^3 e^{2(n+1)}}{(n+1)!5^{n+1}} \cdot \frac{n!5^n}{n^3 e^{2n}} \\
= \left( \frac{n+1}{n} \right)^3 \cdot \frac{e^{2n} \cdot n!}{e^{2n} \cdot (n+1)!} \\
= \left( 1 + \frac{1}{n} \right)^3 \cdot e^2 \cdot \frac{1}{(n+1)}. 
\]

Take the limit of the product by taking the product of the limits (provided they are finite):

\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^3 \cdot e^2 \cdot \frac{1}{(n+1)} = 1 \cdot e^2 \cdot 0 = 0. 
\]

Therefore, by the ratio test, the series converges.