Example

Consider the series
\[ \sum_{n=1}^{\infty} \frac{(-1)^n n^3}{8n^4 + 3}. \]

Use the ratio test to determine if the series converges absolutely.

Computing the limit of the ratio,
\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(-1)^{n+1}(n + 1)^3}{8(n + 1)^4 + 3} \cdot \frac{8n^4 + 3}{(-1)^n n^3} \right|
\]
\[
= \lim_{n \to \infty} \left| \frac{(-1)(n + 1)^3}{n^3} \cdot \frac{8n^4 + 3}{8(n + 1)^4 + 3} \right|
\]
\[
= \lim_{n \to \infty} \left| \frac{(n + 1)^3}{n^3} \cdot \frac{8n^4 + 3}{8(n + 1)^4 + 3} \right|
\]
\[
= \frac{1}{8}. \]

Since \( L = 1 \), the ratio test is inconclusive for determining absolute convergence.

To determine if the series converges absolutely, we instead consider directly
\[ \sum_{n=1}^{\infty} \left| (-1)^n \frac{n^3}{8n^4 + 3} \right| = \sum_{n=1}^{\infty} \frac{n^3}{8n^4 + 3}. \]

Using the limit comparison test with the series \( \sum_{n=1}^{\infty} \frac{1}{n} \), we conclude that the series does NOT converge absolutely.